Int. J. Multiphase Flow Vol. 9, No. 4, pp. 455–457, 1983
Printed in Great Britain.

0301-9322/83 $3.00 + .00
Pergamon/Elsevier

BRIEF COMMUNICATION

REPLY TO COMMENTS ON "VERIFICATION OF MULTICOMPONENT MASS TRANSFER MODELS FOR CONDENSATION INSIDE A VERTICAL TUBE"

D. R. WEBB and R. G. SARDESAI†

Department of Chemical Engineering, University of Manchester Institute of Science and Technology, Manchester M60 1QD, England

(*Received* 15 *February* 1983)

In the preceding note, Krishna has raised an objection to the use of the generalisation of Chilton–Colburn to multicomponent systems. While his new analysis provides a physically more realistic model of multicomponent mass transfer, it must be stated that its predictions will not be significantly different from those of Chilton–Colburn in processes where the Prandtl and Schmidt Numbers are near unity, true of our experimental work. The conclusions of our previous paper are not invalidated though in one important aspect there will be an advantage in using the new analogy.

In our experimental study we were encouraged in the use of the Chilton–Colburn analogy by the excellent agreement of measured and reported correlations of heat transfer coefficients. Thus the best fit (least squares) to extensive measurements of heat transfer coefficients in binary condensation was found to be

$$J_H = 0.039\,\mathrm{Re}^{-0.24}.$$

Now the full form of the Chilton–Colburn analogy is,

$$J_H = \frac{f}{2} = \frac{(h_G)_{cc}\mathrm{Pr}^{2/3}}{C_p G_G/S} = J_D = \frac{(k_{ij})_{cc}\mathrm{Sc}_{ij}^{2/3}}{G_G/S}.$$

Hence our measurements are in almost exact agreement with the Blasius formula,

$$f = 0.0791\,\mathrm{Re}^{-0.25}.$$

The accepted form of the Von Karman equation is now used to estimate the discrepancy between the two analogies over the range of Prandtl and Schmidt numbers normally encountered in vapours and gases at a pressure near 1 atmosphere.

$$\frac{(k_{ij})_{VK}\mathrm{Sc}_{ij}^{2/3}}{G_G/S} = \frac{f/2}{[1 + 5\sqrt{(f/2)}\{\mathrm{Sc}_{ij} - 1 + \ln[1 + \frac{5}{6}(\mathrm{Sc}_{ij} - 1)]\}]}.$$

(A similar equation may be written for $(h_G)_{VK}\mathrm{Pr}^{2/3}/C_p G_G/S$ with Prandtl number replacing Schmidt number.) It is necessary to specify the Reynolds number at which the comparison is to be made since the ratio of transfer coefficients predicted by the two analogies will depend on Reynolds number, through the friction factor, f. In our study the gas Reynolds number was varied between 6000 and 20,000, and the mean of the corresponding values of f realised by the Blasius equation was used. The following table shows the ratio of predicted transfer coefficients by the two analogies.

†Present address: Heat Transfer and Fluid Flow Service (HTFS), Atomic Research Establishment, Harwell.

Pr(Sc)	Ratio
1.5	1.051
1.2	1.025
1.1	1.013
1.0	1.000
0.9	0.984
0.8	0.963
0.7	0.937

Ratio is $\frac{(h_G)_{cc}}{(h_G)_{VK}}$

or $\frac{(k_{ij})_{cc}}{(k_{ij})_{VK}}$.

Thus heat transfer coefficients, h_G, and binary mass transfer coefficients, k_{ij}, predicted by the Chilton–Colburn and Von Karman analogies show a discrepancy of less than 5% over the range of Prandtl and Schmidt numbers of our study.

In calculating rates of multicomponent mass transfer it is not the binary mass transfer coefficients, k_{ij}, but the elements of the matrix of mass transfer coefficients, $[K]$, defined in [6] and table 1 of our paper that are important. It is necessary to calculate how the elements of this matrix are affected by discrepancies in the k_{ij}. The diagonal terms K_{ii} are effectively calculated as a sum of k_{ij} and they are accordingly predicted within 5% by the two analogies. The off-diagonal terms, however, are effectively calculated as a difference of the k_{ij}. Since the k_{ij} are usually of similar magnitude the off-diagonal terms are smaller (typically five times smaller) and hence the relative discrepancies are far larger than those of the diagonal terms. Thus, any quantity which is determined primarily by the magnitude of an off-diagonal element of $[K]$ will probably be sensitive to the choice of analogy. The example quoted by Krishna illustrates this point. He quotes $(K_{12}/K_{11})_{cc} = 0.19$ and $(K_{12}/K_{11})_{VK} = 0.15$ at a Re of 18,000. The ratio of off-diagonal to diagonal elements is one fifth and the discrepancy in off-diagonal elements is about 20%.

Let us now consider how the use of the Von Karman analogy might affect the results of our previous paper. Three aspects must be considered. These are the predictions of sensible heat transfer rates from the gas phase, total mass transfer rates and individual constituent transfer rates. These are considered in turn.

The temperature drop of the gas phase (or the sensible heat removed) is directly proportional to the heat transfer coefficient and is not therefore particularly sensitive to a switch to the Von Karman analogy. The results of figure 5 of our paper, where predicted and experimental temperature drops, are compared and will be only marginally affected.

The total mass transfer rate for the system studies in our paper can be shown to be given by,

$$y_3 N_T = J_1 + J_2 = K_{11}\Delta y_1 + K_{12}\Delta y_2 + K_{21}\Delta y_1 + K_{22}\Delta y_2.$$

Irrespective of the relative magnitudes of Δy_1 and Δy_2, either $K_{11}\Delta y_1$ or $K_{22}\Delta y_2$ must dominate and the predicted total flux of condensation will show the same lack of sensitivity to a change of analogy as the heat transfer coefficient. The latent heat provides the principal contribution to the overall heat load, which in consequence will also be insensitive to choice of analogy.

Individual mass transfer rates are given by the following equation,

$$N_i = K_{i1}\Delta y_1 + K_{i2}\Delta y_2 + y_i N_T.$$

Normally the term $K_{ii}\Delta y_i$ or $y_i N_T$ is dominant and the individual N_i will be insensitive to choice of analogy. However, under conditions where one species dominates the process, it is possible for the condensation rate of the other species to be sensitive to the choice of analogy. Suppose constituent 1 is dominant, and we have the conditions

$$\Delta y_1 \gg \Delta y_2 \qquad\qquad y_2 \sim 0.$$

The condensation rate is then determined by the term $K_{21}\Delta y_1$ which is sensitive to choice of analogy as discussed previously. This particular case was singled out in our paper as an example where the Krishna–Standart method was preferred to the effective diffusivity method, figure 3. It must be said, however, that even in this case, the use of Chilton–Colburn was quite satisfactory for the system studied.

In summary, Krishna has made a valid argument that the Von Karman analogy should be preferred to Chilton–Colburn in treating multicomponent mass transfer. However, under the conditions of our previous study it is apparent that predictions are not sensitive to choice of analogy, so that the conclusions of that study are not invalidated.